Multichannel Uplink NOMA Random Access: Selection Diversity and Bistability

Jun-Bae Seo, *Member, IEEE*, Hu Jin, *Senior Member, IEEE*, and Bang Chul Jung, *Senior Member, IEEE*

***Abstract*—This letter investigates uplink random access (RA) systems based on power-domain non-orthogonal multiple access (NOMA) with multichannel. In NOMA RA systems, users transmit their packet with transmit power by inverting their channel gain, i.e., channel inversion, such that the received power of their packet at the base station (BS) can be one of the predefined values. While this facilitates the decoding process based on successive interference cancellation (SIC), it can incur excessive transmit power consumption upon low channel gains. To reduce transmit power (improve energy efficiency), this letter considers multichannel selection diversity, where users choose one channel whose channel gain is the maximum of all. We characterize the stability of multichannel NOMA RA systems with the cusp catastrophe and analyze diversity gain in terms of energy efficiency.**

***Index Terms*—Bistability, non-orthogonal multiple access, successive interference cancellation.**

I. Introduction

NON-ORTHOGONAL multiple access (NOMA) has received much interest from both academia and industry due to its high spectral efficiency in cellular networks [1]. Recently, it has been applied for uplink random access (RA) protocols [2]–[9]. To make use of power-domain NOMA in uplink RA systems, when multiple users contend for the wireless channel to send their packet, they control their transmit power such that the received power at the base station (BS) can be one of certain *predefined* values. We call such an operation channel inversion in this letter. While the packets transmitted by users are superimposed through the channel, the BS decodes the received packets with successive interference cancellation (SIC) technique. One drawback of this NOMA RA system is that users may consume high (possibly infinite) transmit power with channel inversion, since their channel gain could be often close to zero. This may lead to near zero energy efficiency. To overcome this, we consider multichannel NOMA systems so that users can transmit their packet to a channel whose channel gain is the best of all. This is called selection diversity in the literature.

Among the previous work on uplink NOMA systems [2]–[9], Choi in [8], [9] considered multichannel uplnk NOMA RA systems. More precisely, an approximate throughput lower bound of multichannel NOMA RA systems was investigated in [8]. A retransmission strategy with fading channels was considered based on non-cooperative game-theory in [9].

Compared to [8], [9], our work first shows that multichannel NOMA RA systems can suffer from a phenomenon of bistability according to two control parameters: user's packet generation probability and retransmission probability. It is notable that when a system becomes bistable, it hysterically moves back and forth between two states: one state of yielding a high throughput and another with a very low throughput (undesirable state). Such bistability should be avoided since the system may stay at the undesirable state for a long time. Therefore, our contribution is the characterization of the bistability region of multichannel NOMA RA systems with respect to the two control parameters. Furthermore, by exploiting multichannel (selection) diversity, we improve the energy efficiency of NOMA RA systems.

II. System Model

Suppose a wireless communication system, where a BS has M frequency channels, through which a total of N users communicate with the BS. Time is slotted into a constant size. One slot consists of a downlink subslot and an uplink subslot, i.e., time-division duplex (TDD). At the beginning of each uplink subslot, users (re)transmit their packet with a probability μ to one of M channels. The size of a packet is equal to that of the uplink subslot. We assume that users can hold only one packet. In this letter, a user and a packet can be interchangeably used. The users having a packet to (re)transmit is called *backlogged users* and the number of backlogged users is called *backlog size*. We further assume that each user can have a new packet based on a Bernoulli trial with a probability σ at a slot if he does not have a packet.

In realizing a power-domain uplink NOMA RA system, the BS broadcasts a reference signal of each channel over the downlink subslot. With the channel reciprocity of TDD, we assume that the users can estimate the channel gain of each channel from the downlink reference signal for uplink (re)transmission. Let $h^t_{n,m} \in \mathbb{C}$ be the wireless channel coefficient of the m-th channel of the n-th user for $m \in \{1, \ldots, M\}$ and $n \in \{1, \ldots, N\}$ at time slot t. Hereafter, we omit the time index t for notational simplicity. We assume that $h_{n,m}$ obeys an independently and identically distributed (i.i.d.) Rayleigh fading channel. In other words, $h_{n,m} \sim \mathcal{CN}(0, 1)$ is complex Gaussian random variable with zero mean and unit variance for all n and m. If $X \triangleq |h_{n,m}|^2$, X is an exponential random variable with unit mean, i.e., its PDF $f_X(x) = e^{-x}$. Furthermore, we have additive noise $z_m \sim \mathcal{CN}(0, N_0)$. We assume a block fading for each frequency channel of each user;

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that is, each frequency channel coefficient of a user is constant during one slot and changes to a new independent value for the next time slot. Obviously, different users may have different channel coefficients at each slot.

Let $\mathcal{P}_{n,m}$ be the transmit power of the n-th user to the m-th channel. Based on channel inversion, $\mathcal{P}_{n,m}$ is determined as

$$\mathcal{P}_{n,m} = \begin{cases} P_1/|h_{n,m}|^2 & \text{for } |h_{n,m}|^2 \geq \theta_1 \\ P_2/|h_{n,m}|^2 & \text{for } \theta_2 \leq |h_{n,m}|^2 < \theta_1 \\ 0 & \text{for } |h_{n,m}|^2 < \theta_2, \end{cases} \tag{1}$$

where P_1 and P_2 are called target power levels, and θ_1 and θ_2 are two thresholds for controlling the transmit power. This ensures that the receive power at the BS is P_i for $i \in \{1, 2\}$. If $\mathcal{P}_{n,m} = 0$, there is no (re)transmission.

Without multichannel diversity, a backlogged user selects a channel *randomly*. Then, to realize a packet (re)transmission probability of μ, θ_2 should satisfy $\Pr[|h_{n,m}|^2 \geq \theta_2] = \mu$. On the other hand, with multichannel diversity, the user selects one channel whose channel gain is the maximum of all, i.e., $\arg_{m\in[1,M]} \max |h_{n,m}|^2$. Therefore, θ_2 can be controlled to satisfy $\Pr[\max(|h_{n,1}|^2, \ldots, |h_{n,M}|^2) \geq \theta_2] = \mu$. In (1) that $\mathcal{P}_{n,m}$ can take a large value if $\theta_2 \to 0$. However, it is expected that the multichannel diversity increases θ_2 and consequently reduces the transmit power. Note that as in [6], [8], the system can employ multiple levels of target powers. In this work, for convenience, we focus on the system with two power levels P_1 and P_2.

Let u_n and $b_{n,m} \in \{0, 1\}$ be the modulated symbol with unit power (i.e., $\mathbb{E}[|u_n|^2] = 1$) and the transmission indicator for the m-th channel of the n-th user. If N_m is the number of users (re)transmitted to the m-th channel, the signal that the BS receives at the m-th channel is expressed as $\boldsymbol{r}_m = \sum_{n=1}^{N_m} h_{n,m}\sqrt{\mathcal{P}_{n,m}} b_{n,m} u_n + z_m$. With a perfect successive interference cancellation (SIC) technique assumed at the BS, the packet decoding is successful at each channel, if signal-to-interference-noise ratio (SINR) satisfies the conditions $\frac{P_1}{P_2+N_0} \geq \psi$ and $\frac{P_2}{N_0} \geq \psi$. When the minimum powers P_1 and P_2 are considered for the SINR condition, we have $P_1 = N_0\psi(1 + \psi)$ and $P_2 = N_0\psi$. Then, two packets can be successfully decoded at maximum, where one with P_1 and the other with P_2 are (re)transmitted. In addition, if only one packet is (re)transmitted either with P_1 or P_2, it is always successfully decoded. Notice that the (re)transmission outcome such as success or failure is immediately broadcast to the users in the following downlink subslot.

III. Performance Analysis

A. Throughput and Bistability

Lemma 1: Suppose that M-channel NOMA RA system has k backlogged users at a slot and users choose target power P_1 (or P_2) with probability p (or q) *given that they (re)transmit*. Then, the throughput-maximal retransmission probability for users to employ, denoted by μ_k^*, is expressed as

$$\mu_k^* = \min\left(1, M\sqrt{2}/k\right). \tag{2}$$

Proof: Let us denote by τ and $S(k)$ the throughput of a single channel and the overall throughput of M channels when there are k backlogged users. Since the users select one of M channels independently with probability $\frac{1}{M}$, we can write $S(k) = M\tau$. This is valid whether multichannel diversity is applied or not.

To find τ, let μ denote the (re)transmission probability of a backlogged user. For notational convenience, let $\mathcal{B}(i, m, x) \triangleq \binom{m}{i} x^i(1 - x)^{m-i}$. We can write τ as

$$\begin{aligned}\tau &= \sum_{i=1}^{k} \mathcal{B}\left(1, i, \frac{1}{M}\right) \mathcal{B}(i, k, \mu) \\ &\quad + 2\sum_{i=2}^{k} \mathcal{B}\left(2, i, \frac{1}{M}\right) \mathcal{B}(i, k, \mu), \\ &= \frac{k\mu}{M}\left(1 - \frac{\mu}{M}\right)^{k-1} + 2k(k-1)pq\left(\frac{\mu}{M}\right)^2\left(1 - \frac{\mu}{M}\right)^{k-2} \\ &= \frac{k}{M}\left(\mu + \frac{2(k-1)pq - 1}{M}\mu^2\right)\left(1 - \frac{\mu}{M}\right)^{k-2},\end{aligned} \tag{3}$$

where the first term in the first line is that a backlogged user retransmits successfully regardless of target power P_1 or P_2 if he is the only (re)transmitting user in a channel. The second term shows that two backlogged users make successful retransmissions, when one chooses target power P_1 and the other one does target power P_2.

When μ_k^* denotes the maximizer of τ, it can be obtained by $\frac{d\tau}{d\mu} = 0 \Rightarrow -\frac{k}{M^2}(2(k-1)pq - 1)\mu^2 - \frac{k(1-4pq)+1+4pq}{M}\mu + 1 = 0$. We then get $\mu_k^* = \frac{M\sqrt{2}}{k}$. Since $\mu_k^* \leq 1$, we have (2). ∎

The following remarks can be made: ***First***, plugging (2) into (3), we have $\tau = (\sqrt{2} + 4pq)e^{-\sqrt{2}}$. When k backlogged users employ (2), we denote by $\hat{S}(k)$ the throughput achieved with (2). It can be obtained as

$$\hat{S}(k) = M\tau = M(\sqrt{2} + 4pq)e^{-\sqrt{2}}, \tag{4}$$

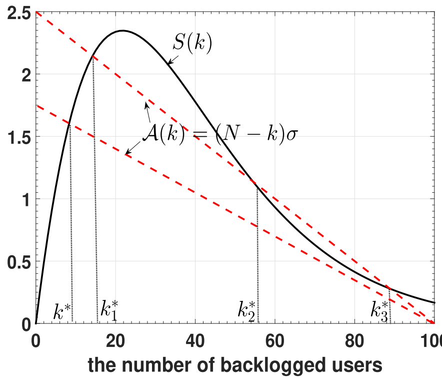
which is independent of k. To achieve $\hat{S}(k)$ and/or realize (2), either the BS or the backlogged users should know the number of backlogged users k, or at least estimate it. *Second*, since p and q are the probability that a user chooses target power P_1 or P_2 *provided that he (re)transmits*, it follows that $p + q = 1$. This shall be further examined with selection diversity scheme in Section III-B. We can maximize $\hat{S}(k)$ at $p = 1/2$, which is $\hat{S}(k) = 0.5869M$ (linearly proportional to M).

Proposition 1: Recall that the system has a total of N users, each of whom produces a packet with probability σ. Let $\mathcal{A}(k)$ denote the average number of users (or packets) that newly join the backlogged users when the system has k backlogged users at a slot. Since a packet is generated with probability σ at a slot, it follows that $\mathcal{A}(k) = \sum_{i=0}^{N-k} \mathcal{B}(i, N-k, \sigma) = (N-k)\sigma$ (packets/slot). The system is stable if there exists a unique k^* such that

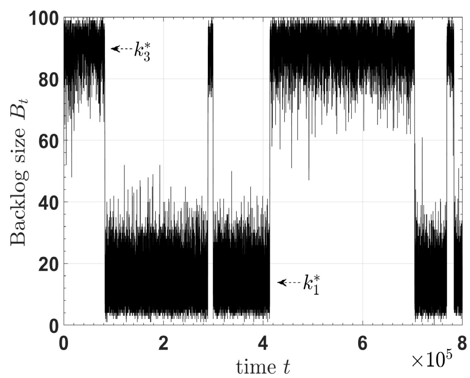
$$S(k^*) = \mathcal{A}(k^*) \quad \text{for} \quad k^* \in [0, N]. \tag{5}$$

Proof: Let S_t, A_t, and B_t denote the number of packets successfully (re)transmitted, newly arriving packets, and the number of backlogged users at slot t, respectively. Then, the evolution of backlogged users at the next slot is expressed as

$$B_{t+1} = B_t - S_t + A_t \quad \text{for} \quad S_t \leq B_t. \tag{6}$$



(a) Input and throughput relation



(b) Sample path of backlog size in bistable system

Fig. 1. (Bi)Stability of uplink NOMA system.

As $t \to \infty$, a finite $\mathbb{E}[B_\infty]$ should exist in the steady-state. If so, it can be seen that $\lim_{t\to\infty} \mathbb{E}[B_{t+1}] = \lim_{t\to\infty} \mathbb{E}[B_t - S_t + A_t]$ exists as well. We then have $\mathbb{E}[S_\infty] = \mathbb{E}[A_\infty]$. Notice that $\mathbb{E}[S_\infty | B_\infty = k] = S(k)$ and $\mathbb{E}[A_\infty | B_\infty = k] = \mathcal{A}(k) = (N-k)\sigma$. When $\mathbb{E}[B_\infty] = k^*$ in the steady-state, (5) should hold. ∎

Proposition 1 implies that the average input $\mathcal{A}(k)$ should be equal to the throughput $S(k)$ for a stable system, which occurs at a single state k^*. Let us further consider the bistability of this system with Proposition 1. In Fig. 1(a), the average number of backlogged users for a stable system is depicted by k^*, where we set $\mu = 0.26$, $\sigma = 0.0175$, $N = 100$, and $M = 3$. It is important to note that once N and M are given, depending on μ and σ, e.g., σ increased up to 0.025, there can be three points (or states) such as k_1^*, k_2^*, and k_3^*, at which $S(k)$ and $\mathcal{A}(k)$ meet. The k_1^* and k_3^* are then called *locally equilibrium points (states)*. In this case, the system is called *bistable*. In the bistable system, the backlog size shows an oscillating behavior as shown in Fig. 1(b); that is, it bounces back and forth between $k_1^* \approx 13$ and $k_3^* \approx 90$ over time. In other words, the system shows a high throughput at state k_1^*, while nearly zero throughput at state k_3^*. It is possible that a small increase in the input rate introduces a sudden increase (or change) in the backlog size such that the system could stay in such a large (backlog) state with zero throughput for a long time. Accordingly, such a bistable system should be avoided in order to prevent the system from unexpected collapses. Recall that if (2) can be realized, the system achieves $\hat{S}(k)$ in (4). From $\hat{S}(k^*) = \mathcal{A}(k^*)$, we then have a stable system, i.e., a unique state $k^* = \max(0, N - M(\sqrt{2} + 4pq)e^{-\sqrt{2}}/\sigma)$. However, this requires knowledge on k every slot t. We now characterize what parameters can cause such bistability.

Before carrying out bistability analysis, let us return to (5). It can be seen that k^* is an integer in (3) so that an exact equality $S(k^*) = \mathcal{A}(k^*)$ is not achieved. However, $E[B_\infty] = k^*$ in simulation takes a real number. Thus, to get an accurate (real-valued) k^*, we consider a normalized backlog size $x \triangleq \frac{k}{N}$ and $G = Nx$. Once $x^* \in [0,1]$ is found, a real-valued $k^* = Nx^*$ is obtained as well. By assuming that the number of backlogged users follows a Poisson process with mean $Nx = G$, we write $\overline{S}_x = \sum_{k=0}^{\infty} S(k)\frac{(Nx)^k}{k!}e^{-Nx} = \left(\mu G + 2pq\frac{(\mu G)^2}{M}\right)e^{-\frac{\mu G}{M}}$, and $\overline{A}_x = N(1-x)\sigma$. Then, analogous to (5), the normalized backlog size x^* can be found by solving $\mathcal{F}(x) = \overline{S}_x - \overline{A}_x = 0$. By setting $\mathcal{G} \triangleq Nx\mu = \mu G$, we can write it as

$$\mathcal{F}(x) = \mathcal{G}\left(1 + 2pq\frac{\mathcal{G}}{M}\right)e^{-\frac{\mathcal{G}}{M}} - \left(N - \frac{\mathcal{G}}{\mu}\right)\sigma = 0. \quad (7)$$

When N and M are given in (7), the characterization of the bistability of multichannel NOMA RA systems is to find a set of two (control) parameters σ and μ that drive the system bistable. To do this, we apply the catastrophe theory [11]: Let $\mathcal{F}$ be a potential function of the system, and $\mathcal{F}: \mathbb{R}^k \times \mathbb{R}^n \to \mathbb{R}$, where k and n are the number of control variables and system states, respectively. If $k = 2$, we have the fold and the cusp catastrophes and $n = 1$, i.e., the system state x. Then, the catastrophe manifold Θ is a surface in three dimensions defined by $\Theta = \{(x,\sigma,\mu)|\mathcal{F}(x) = 0\}$. Let us further denote by Θ_B the fold line consisting of the points of Θ, where the manifold surface folds over. It is defined as $\Theta_B = \left\{(x,\sigma,\mu)|\mathcal{F}(x) = \frac{d\mathcal{F}(x)}{dx} = 0\right\}$. Using $\frac{d^2\mathcal{F}(x)}{dx^2}$, we can characterize Θ_B as three parts:

$$\Theta^+ = \left\{(x,\sigma,\mu)|\mathcal{F}(x) = \frac{d\mathcal{F}(x)}{dx} = 0, \frac{d^2\mathcal{F}(x)}{dx^2} > 0\right\}, \quad (8)$$

$$\Theta^- = \left\{(x,\sigma,\mu)|\mathcal{F}(x) = \frac{d\mathcal{F}(x)}{dx} = 0, \frac{d^2\mathcal{F}(x)}{dx^2} < 0\right\}, \quad (9)$$

and $\Theta_0 = \left\{(x,\sigma,\mu)|\mathcal{F}(x) = \frac{d\mathcal{F}(x)}{dx} = \frac{d^2\mathcal{F}(x)}{dx^2} = 0\right\}$ called the cusp point. The bifurcation sets B^+ and B^- are obtained as the projection of the three-dimensional catastrophe manifold (fold line) Θ^+ and Θ^- into the control space (σ,μ), respectively. The following proposition finds σ and μ for B^+ and B^-.

Proposition 2: For B^+ and B^-, when μ is given, there are two σ's that are characterized by

$$\sigma = \frac{2pq\mathcal{G}^2}{NM}\left(1 + \frac{\mathcal{G}}{M}\right)e^{-\frac{\mathcal{G}}{M}}, \quad (10)$$

where if $b^2 + 4a^3 \le 0$ ($a = -(M-N\mu)^2/9$ and $b = 2(\frac{1}{27}(M-N\mu)^3 + \mu NM^2)$), $\mathcal{G}$ takes two positive real values as

$$\mathcal{G} = \begin{cases} 2\sqrt[3]{h}\cos(\theta/3) - \dfrac{M-N\mu}{3} & \text{for } B^+ \\ 2\sqrt[3]{h}\cos((\theta+4\pi)/3) - \dfrac{M-N\mu}{3} & \text{for } B^-. \end{cases} \quad (11)$$

In (11), it follows that $\theta = \cos^{-1}\left(-\frac{b}{2h}\right)$ and $h = \sqrt{-a^3}$.

Proof: We get $\frac{dF(x)}{dx} = N\sigma + N\mu(1-2pq\,(G/M)^2)e^{-\frac{\mu\mathcal{G}}{M}}$. From $\frac{dF(x)}{dx} = 0$, it can be rearranged as

$$\sigma = -\mu(1 - 2pq(\mathcal{G}/M)^2)e^{-\mathcal{G}/M}. \tag{12}$$

Plugging (12) into (7) yields

$$\mathcal{G}^3 + (M - N\mu)\mathcal{G}^2 + \mu N M^2/(2pq) = 0. \tag{13}$$

We need to find the roots of the cubic equation in (13). In [10], it can be found that (13) can have two positive real roots if $b^2 + 4a^3 < 0$, which are given in (11); one real positive root if $b^2 + 4a^3 = 0$; otherwise, complex roots, which are physically invalid for our system. On the other hand, by rewriting (7) with respect to μ and plugging this in μ of (12), we have (10). Thus, at a given μ, we get two $\mathcal{G}$'s with (11), which produce two σ's such that B^+ and B^- can be found. ∎

Proposition 3: The cusp point (σ^*, μ^*) is determined by substituting $\mathcal{G}^* = \frac{M\left(8pq-1+\sqrt{32(pq)^2+1}\right)}{4pq}$ into (10) and (12). Particularly for $p = q = 0.5$, we have $\mathcal{G}^* = M(1+\sqrt{3})$.

Proof: By definition of the cusp point, we examine

$$\frac{d^2\mathcal{F}(x)}{dx^2} = \frac{(N\mu)^2\, e^{-\frac{\mathcal{G}}{M}}}{M^3}\big(2M^2(2\,pq-1) + \mathcal{G}(M(1-8pq)+2pq\mathcal{G})\big) = 0,$$

whose positive root is $\mathcal{G}^*$. Notice that this $\mathcal{G}^*$ is the solution of (11) for $b^2 + 4a^3 = 0$ such that two solutions in (11) are equal. ∎

B. Energy Efficiency With Diversity

Let us examine energy efficiency, denoted by $\overline{E}$. If $\overline{\mathcal{P}}_i$ denotes the average power consumption with target power P_i, the information theoretic energy efficiency is obtained as

$$\overline{E} = \log(1+\psi)S(k^*)/(\overline{\mathcal{P}}_1 + \overline{\mathcal{P}}_2) \tag{14}$$

with some μ_k^*, and k^* is the average backlog size. Let α (or β) denote the probability that a backlogged user (re)transmits with target P_1 (or P_2) to one of M channels. Without selection diversity, we can write

$$\alpha = \Pr[|h_{n,m}|^2 \geq \theta_1] = 1 - F_X(x) = e^{-\theta_1}. \tag{15}$$

On the other hand, we have

$$\begin{aligned}\beta &= \Pr[\theta_2 \leq |h_{n,m}|^2 < \theta_1] = F_X(\theta_1) - F_X(\theta_2)\\ &= e^{-\theta_2} - e^{-\theta_1}.\end{aligned} \tag{16}$$

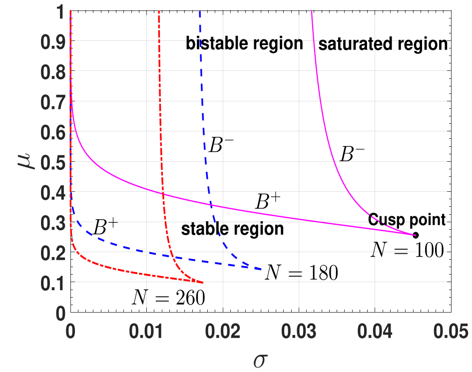
To find θ_2, from (2), we have $\mu_k = \alpha + \beta$. This yields

$$\theta_2 = -\ln\min(1, M\sqrt{2}/k). \tag{17}$$

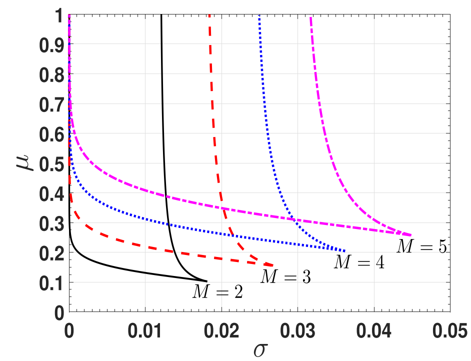
In (3), it follows that $p = \alpha/(\alpha+\beta)$ and $q = \beta/(\alpha+\beta)$. If $\alpha = \beta$, we have $p = q = 0.5$, which maximizes (3). Thus, once θ_2 is found, from $\alpha = \beta$, we can determine $\theta_1 = \ln 2 + \theta_2$. In (14), we get

$$\overline{\mathcal{P}}_1 = \frac{P_1}{\alpha}\int_{\theta_1}^{\infty}\frac{1}{x}e^{-x}dx = \frac{P_1}{\alpha}E_1(\theta_1), \tag{18}$$

where the exponential integral $E_1(x) = \int_x^\infty \frac{e^{-y}}{y}dy$, and $\overline{\mathcal{P}}_2 = \frac{P_2}{\beta}\int_{\theta_2}^{\theta_1}\frac{1}{x}e^{-x}dx = \frac{P_2}{\beta}(E_1(\theta_2) - E_1(\theta_1))$.



(a) The Bifurcation sets with $N = 100, 180$ and $M = 5$.



(b) The Bifurcation sets with various M's and $N = 100$.

Fig. 2. The Bifurcation set of multichannel NOMA RA system.

With multichannel diversity, backlogged users select one channel whose SNR is the maximum of M channels. From $\Pr[\max(|h_{n,1}|^2, \ldots, |h_{n,M}|^2) < x] = (1-e^{-x})^M$, we have

$$\begin{aligned}\alpha &= \Pr[\max(|h_{n,1}|^2, \ldots, |h_{n,M}|^2) \geq \theta_1]\\ &= 1 - (1-e^{-\theta_1})^M\\ \beta &= (1-e^{-\theta_1})^M - (1-e^{-\theta_2})^M.\end{aligned} \tag{19}$$

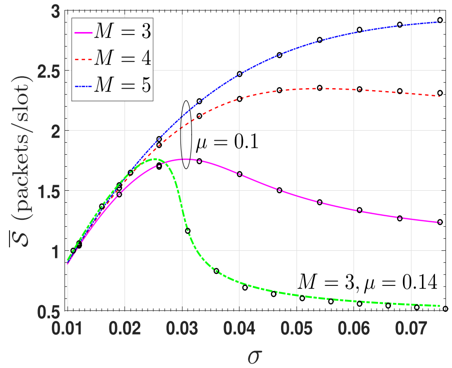
For the system with diversity, from $\mu_k = \alpha + \beta$, the threshold for (re)transmission is determined as

$$\theta_2 = -\ln(1 - (1 - \min(1, M\sqrt{2}/k))^{\frac{1}{M}}), \tag{20}$$

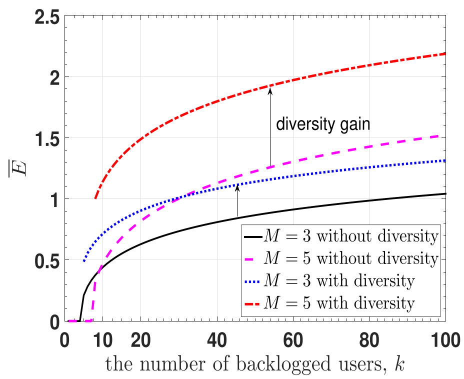
and $\theta_1 = -\ln\left(1 - \left(0.5(1 + (1-e^{-\theta_2})^M)\right)^{\frac{1}{M}}\right)$ from $\alpha = \beta$. We can calculate the average transmit power with $\overline{\mathcal{P}}_1 = \frac{P_1}{\alpha}\mathcal{K}(\theta_1)$ and $\overline{\mathcal{P}}_2 = \frac{P_2}{\beta}(\mathcal{K}(\theta_2) - \mathcal{K}(\theta_1))$, where $\mathcal{K}(z)$ is expressed as

$$\begin{aligned}\mathcal{K}(z) &= \int_z^\infty \frac{1}{x}M(1-e^{-x})^{M-1}e^{-x}dx\\ &= M\sum_{i=0}^{M-1}\binom{M-1}{i}(-1)^i\int_z^\infty\frac{1}{x}e^{-(i+1)x}dx\\ &= M\sum_{i=0}^{M-1}\binom{M-1}{i}(-1)^i E_1((i+1)z).\end{aligned}$$

Note that the throughput analysis in Proposition 1 is valid since one channel is selected with probability $1/M$. In addition, whether the diversity is applied or not, the energy



(a) Throughput with various M's.



(b) Energy efficiency with M

Fig. 3. Performance of multichannel NOMA RA systems.

efficiency is a decreasing function of ψ. To see this, from (14) we have

$$\overline{E} = \frac{\log(1+\psi)S(k^*)}{N_0\left(\frac{\psi(1+\psi)}{\alpha}E_1(\theta_1) + \frac{\psi}{\beta}(E_2(\theta_2) - E_1(\theta_1))\right)}. \quad (21)$$

As $\psi \to \infty$, we can see that $\overline{E}$ goes to zero. Thus, ψ is needed as much as the SINR threshold can be met.

IV. Numerical Results

In Fig. 2(a), the bifurcation sets are presented for $M = 5$. For a given σ, the system has a small backlog size for μ to the left of B^+ and/or below B^+, marked as stable region. On the other hand, for μ and σ to the right of B^-, the backlog size gets close to N, i.e., in saturated region with very low throughput. The system becomes bistable if μ and σ are chosen in bistable region. For example, the parameters used in Fig. 1(b), $\sigma = 0.0175$, $\mu = 0.26$, $N = 100$, and $M = 3$ are found in bistable region. As N increases, stable region of σ and μ gets smaller. This means that the system gets saturated for most of σ and μ. In Fig. 2(b), the bifurcation sets are shown for $N = 100$ and various M's. As M increases, it can be seen that the stable region, which is a set of σ and μ with desirable throughput, gets larger.

Fig. 3(a) shows the system throughput $\overline{\mathcal{S}} = S(x^*)$ with $p = q = 0.5$, where symbol $\circ$ indicates simulation results. The analysis looks quite accurate; that is, the true behavior of the backlog size may be insensitive to the Poisson assumption (with mean Nx). While $\overline{\mathcal{S}}$ is improved as M increases, it can be observed that the maximum throughput corresponds to $0.5869M$. Referring to Fig. 2(b), if $M = 3$, we can see the stable region for $\sigma < 0.02718$ with $\mu = 0.1$. In Fig. 3(a), $\overline{\mathcal{S}}$ (with $M = 3$) approaches to its maximum as σ goes to 0.02718. Note that when μ is increased to 0.14 and as σ exceeds 0.02718 (or B^-), the system becomes saturated in Fig. 2(b). This corresponds to significant drop in $\overline{\mathcal{S}}$ in Fig. 3(a) with $\mu = 0.14$ as σ exceeds 0.02718. In Fig. 3(b), we examine the energy efficiency $\overline{E}$ with $\psi = 2$ and $N_0 = 1$. As M increases (more channels used), it can be seen that θ_2 decreases in (17) and (20). This increases the retransmission opportunities and consequently increases the throughput.. Furthermore, it can be found that the larger M, the higher $\overline{E}$. For small k, where $\mu_k = 1$ and $\theta_2 \approx 0$, we get $\overline{E} = 0$, which is not depicted in Fig. 3(b). It can be also seen that $\overline{E}$ with multichannel diversity for $M = 3$ can be even better for some k than that without diversity for $M = 5$.

V. Conclusion

This work examined the bistability of multichannel NOMA RA systems and improved the energy efficiency by exploiting multichannel diversity. More precisely, the bistability region of this system was characterized in terms of the bifurcation sets with the packet generation probability σ and the retransmission probability μ. Thus, the bistability can be avoided in practice by not using the variables causing it. We also showed that as the number of channels increases, the stability and bistability regions get larger, while the energy efficiency was improved with multichannel diversity.

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